COUPLING OF CYLINDRICAL DIELECTRIC RESONATORS TO MICROSTRIP LINES*

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ABSTRACT

A lumped element circuit model is introduced to represent coupling between a cylindrical dielectric resonator and a microstrip line. The external Q of the structure is computed and compared to experimental data obtained with three different resonators.

Introduction

The recent availability of low-loss, temperature-stable dielectric materials has encouraged the development of several microwave devices employing high dielectric constant resonators. Among the explored applications are temperature-compensated oscillators,[1] low-noise microwave synthesizers,[2] and narrow bandpass filters.[3] These new devices utilize cylindrical dielectric resonators coupled to a transmission line which is generally in microstrip configuration. However, to the authors' best knowledge, the open literature does not contain a theoretical approach that deals with the quantitative computation of the coupling between a microstrip line and a dielectric resonator.

This paper presents an approximate lumped element circuit model to describe this phenomenon and to compute the fundamental parameters, based on previously derived field theory.[4] The external Q of the cavity composed by the line and resonator is computed and the result is compared to three independent sets of measured data, showing good agreement between theory and experiment.

Geometry Under Analysis and Basic Assumptions

A cross section and top view of the geometry under analysis are shown in Figure 1, and three examples of practical configurations are provided in Figure 2. The basic assumptions are as follows:

a. Only the dominant mode ($TE_{10\delta}$) is present in the structure.
b. The microstrip line is considered as a small perturbation in the field distribution inside the cavity, and its effects are neglected.
c. The length of the circular section of the microstrip is smaller than a quarter wavelength in the substrate.
d. The dielectric constants involved are such that $\varepsilon_3 > \varepsilon_i$ (i = 1, 2, 4, 5) and $\varepsilon_1 >> \varepsilon_2$.
e. All losses involved are small enough to be neglected.

For all practical purposes, these assumptions do not create major constraints in the degrees of freedom the structure offers.

Circuit Analysis

One possible lumped element circuit configuration that represents a resonator magnetically coupled to a transmission line is shown in Figure 3, where the series resonant circuit represents the dielectric resonator and the pi-circuit a short microstrip line. The coupling of energy occurs only through the mutual inductance "m" between the circuits (radiation effects are neglected based on assumption d). The circuit elements (L_p, C_p) are the total inductance and capacitance of the microstrip line section as perturbed by the presence of the high constant dielectric material; L_r and C_r are such that the resonant frequency of the overall structure, (ω_r), is given by $\omega_r = (L_r C_r)^{-1/2}$.

The input impedance of the circuit in Figure 3 is readily computed as

$$Z_{in} = j\,\frac{1 + \left(\omega L_p - \dfrac{2}{\omega C_p}\right)\dfrac{2\omega_r L_r \delta}{\omega^2 m^2}}{\dfrac{4\omega_r L_r \delta}{\omega^2 m^2} + \dfrac{\omega C_p}{2} - \dfrac{\omega_r L_r \delta}{m^2} L_p C_p} \qquad (1)$$

where

$$\delta = \frac{(\omega - \omega_r)}{\omega_r} \qquad (2)$$

The external Q, as defined by

$$Q_e = \frac{\omega_r}{2Z_o} \left.\frac{\delta X_{in}}{\delta \omega}\right|_{\omega=\omega_r} \qquad (3)$$

can be computed from equation (1), yielding

$$Q_e = \frac{4Z_p^2}{Z_o Z_c} + \frac{Z_p}{Z_o} \qquad (4)$$

with

$$Z_p = (\omega_r C_p)^{-1} \qquad (5)$$

$$Z_c = \frac{\omega_r m^2}{L_r} \qquad (6)$$

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The parameter Z_c is hereafter referred to as "coupling impedance." Note that Q_e has a lower bound, Z_p/Z_o, and therefore will not drop indefinitely with increasing values of Z_c.

Field Analysis

The coupling impedance is computed based on the information given about field components in Reference 4. The self-inductance of the dielectric resonator, as a function of the loop current of the equivalent circuit and of the stored magnetic energy (peak value), is defined by

$$L_r = \frac{W_m}{I_r^2} \quad (7)$$

Under resonant conditions, the stored magnetic energy can be computed from the stored electric energy as

$$W_m = W_e = \frac{1}{2} \iiint_v \varepsilon E^2 \, dv \quad (8)$$

The voltage drop induced in the microstrip due to a current, I_r, in the resonant loop is

$$\Delta V = j\omega m I_r \quad (9)$$

and can also be computed from the magnetic flux in loop ABCD (Figure 4) as

$$\Delta V = j\omega\mu_o \iint_{S_1} \underline{H} \cdot d\underline{S}_1 \quad (10)$$

Combining equations (7) through (10) and substituting into equation (6) yield

$$Z_c = \frac{\omega_o \mu_o^2 \left(\iint_{S_1} \underline{H} \cdot d\underline{S}_1 \right)^2}{\frac{1}{2} \iiint_v \varepsilon E^2 \, dV} \quad (11)$$

Numerical and Experimental Results

Figure 5 shows the dependence of the normalized external Q on the position of the microstrip coupling loop; the maximum coupling position predicted at $R_o/R_1 = 0.65$ is confirmed by two independent experiments. The test jigs were etched with 50Ω microstrip lines over a 0.050-in.-thick alumina substrate; all lines were of equal length but varying radius. Figure 6 shows the dependence of the external Q, as computed from equation (4), on the line length using as a parameter the ratio between the perturbed and unperturbed value of the total line capacitance. The slope of the experimental data is in good agreement with the theory for small line lengths, as expected from the simple lumped element model used in the microstrip representation. The perturbation introduced by the presence of the dielectric resonator over the microstrip is not negligible; in this case, the 50Ω line was reduced to about 43Ω. Figure 7 exhibits the same type of data, measured with a different resonator, and also leads to similar conclusions.

Conclusions

A simple lumped element circuit model is proposed that represents a dielectric resonator coupled to a microstrip line. The external Q of the circuit is computed from previously derived field theory,[4] and shows good agreement with experimental data.

References

1. J. K. Plourde et al., "A Dielectric Resonator Oscillator with 5 ppm Long Term Frequency Stability at 4 GHz," IEEE MTT-S International Microwave Symposium Digest, June 1977, pp. 273-276.

2. G. D. Alley and H. C. Wang, "An Ultra Low-Noise Microwave Synthesizer," Digest of the 1979 International Microwave Symposium, Orlando, Florida, pp. 147-149.

3. J. K. Plourde and D. F. Linn, "Microwave Dielectric Resonator Filters Utilizing $Ba_2Ti_2O_{20}$ Ceramics," IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-27, March 1979, pp. 233-238.

4. R. Bonetti and A. Atia, "Design of Cylindrical Dielectric Resonators in Inhomogeneous Media," IEEE Transactions on Microwave Theory and Techniques, scheduled for April 1981.

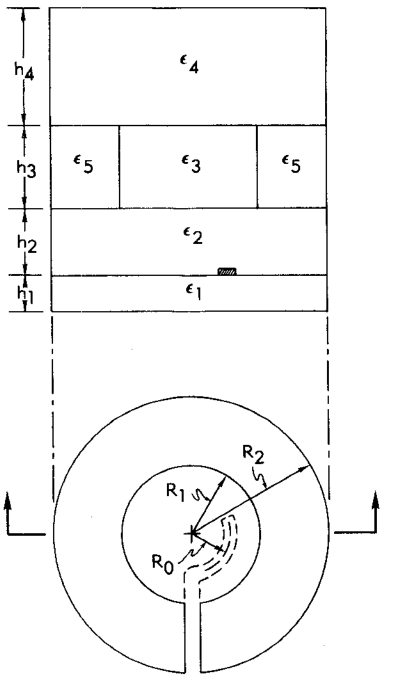

Figure 1. Cross Section and Top View of Geometry Under Analysis

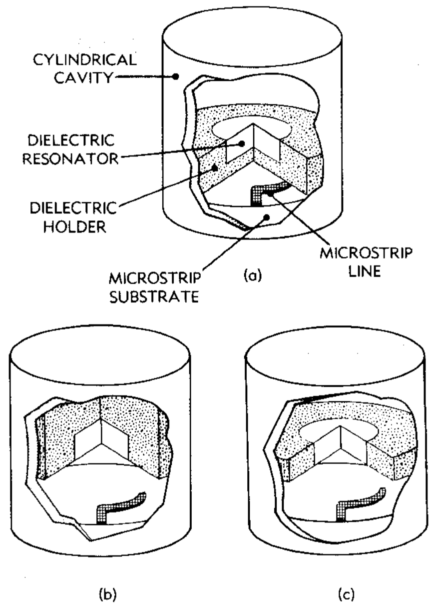

Figure 2. Practical Coupled Resonator Configurations

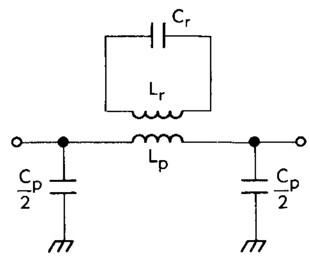

Figure 3. Lumped Equivalent Circuit

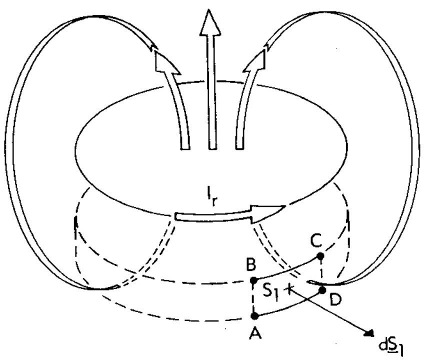

Figure 4. Magnetic Flux Linkage

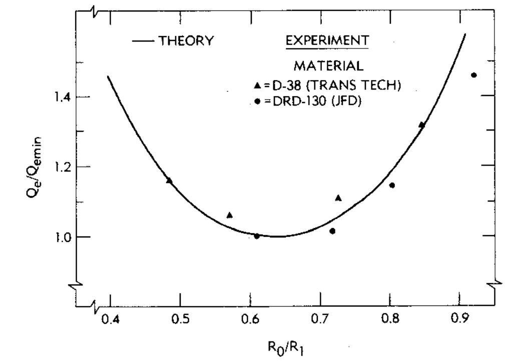

Figure 5. Normalized External Q as a Function of Micro-strip Position

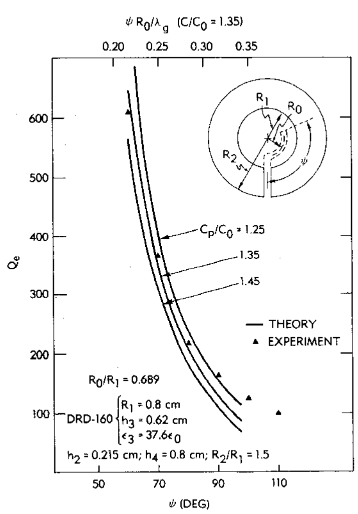

Figure 6. External Q as a Function of Microstrip Coupling Angle

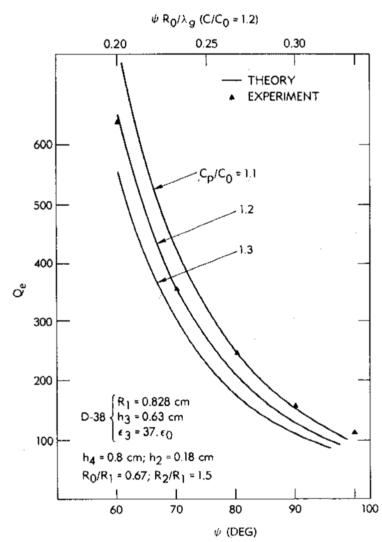

Figure 7. External Q as a Function of Microstrip Coupling Angle (2nd experiment)